



Missouri University of Science and Technology
Scholars' Mine

International Conference on Case Histories in
Geotechnical Engineering

(2013) - Seventh International Conference on
Case Histories in Geotechnical Engineering

02 May 2013, 4:00 pm - 6:00 pm

WSASW Application for Soil Dynamic Properties in Soft Soil Investigation at Kelang, Malaysia

Sri Atmaja P. Rosyidi
Universitas Muhammadiyah Yogyakarta, Indonesia

Follow this and additional works at: <https://scholarsmine.mst.edu/icchge>

 Part of the [Geotechnical Engineering Commons](#)

Recommended Citation

Rosyidi, Sri Atmaja P., "WSASW Application for Soil Dynamic Properties in Soft Soil Investigation at Kelang, Malaysia" (2013). *International Conference on Case Histories in Geotechnical Engineering*. 42.
https://scholarsmine.mst.edu/icchge/7icchge/session_06/42

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in International Conference on Case Histories in Geotechnical Engineering by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.

WSASW APPLICATION FOR SOIL DYNAMIC PROPERTIES IN SOFT SOIL INVESTIGATION AT KELANG, MALAYSIA

Sri Atmaja P. Rosyidi

Universitas Muhammadiyah Yogyakarta
Yogyakarta, Indonesia, 55183

ABSTRACT

In seismic wave measurements, soil dynamic parameters, i.e., shear wave velocity and shear modulus are determined from a dispersion curve of phase velocity with employing the theory of elasticity of soils. In this paper, a developed technique from Spectral Analysis of Surface Waves (SASW), namely as the Wavelet Spectrogram Analysis of Surface Waves (WSASW) is introduced for measurement of the soil dynamic properties at soft marine clay soils sites. This technique has capability to localize the interested response spectrum of surface waves, to reconstructed spectrograms of noisy seismic waves and produces the enhanced phase data to develop the phase velocity dispersion curve. A filtration procedure is also proposed in order to remove the noisy signals from the seismic records received during field measurement. The identification, denoising and reconstruction of the wave response spectrum from seismic surface wave propagation on a soft soil were carried out by using time-frequency analysis of continuous wavelet transforms. The wavelet analysis was used for providing good resolution of spectrogram at low frequency and their spectrograms could be used to clearly identify the various events of interest of the seismic surface waves and noisy signals. Good agreements were obtained between the measured shear wave velocities and the corresponding dynamic shear modulus by the WSASW technique compared to Continuous Surface Wave (CSW), SASW and conventional soil testing method that performed in the same site location. Some good empirical correlations were also obtained as study case for soft soil site.

INTRODUCTION

Soil dynamic properties of shear wave velocity (V_s) and shear modulus (G) are important parameters in geotechnical earthquake engineering problems associated with dynamic loading at low to moderate-strain levels. These parameters can be in situ evaluated by using seismic methods, i.e., measurement of wave velocities propagating through soil medium. The spectral analysis of surface wave (SASW) is one of popular seismic methods used for this purpose. Much of the basis of the theoretical and analytical work of this method for soil investigation has been developed (Stokoe et al. 1994). However, the phase velocity curve is analyzed from huge amounts of non-stationary seismic data in nature i.e. varying frequency content in time. Therefore, the time-frequency localization is needed to provide accurate information of wave spectrum. In conventional surface wave method, the data analysis in both time and frequency domain has been carried out fast Fourier transforms for various system responses in different frequency from several input motions with time. Due to Fourier transform works by expressing any arbitrary periodic function of time with period as sum a set of sinusoidal, some information of non-stationary seismic data in

analysis maybe lost. In addition, the Fourier analysis is unable to preserve the time dependence and describe the evolutionary spectral characteristics of non-stationary processes.

Consequently, a new tool of wavelet analysis is required employing in surface wave method which allows time and frequency localization of the signals beyond customary Fourier analysis. Wavelet analysis is becoming a common tool for analyzing localized variations of power within a time series. The wavelet transform has been used in numerous studies in geophysics, geotechnical engineering and site investigation. A complete description of geophysical applications can be found in Foufoula-Georgiou & Kumar (1995), while a theoretical treatment of wavelet analysis is given in Daubechies (1992).

The objective of this paper is to present the capability of the Wavelet Spectrogram Analysis of Surface Waves (WSASW) based on mother wavelet of Morlet in order to evaluate the shear wave velocity and shear modulus of soil site. In this method, the time-frequency wavelet (TFW) spectrum has been

employed to localize the interested response spectrum of surface waves of the soil profile. Results from field studies are presented for in situ evaluation of soil dynamic parameters, i.e., the shear wave velocity and the dynamic shear modulus.

RESEARCH METHOD

Continuous Wavelet Transform

The continuous wavelet transform (CWT) technique is becoming a common tool to analyse localised variation of power within a time series for non-stationary signal i.e. seismic signals. In the technique, wavelets dilate in such a way that the time support also changes for different frequency. When the time support increases or decreases, the frequency support of the wavelet is shifted towards high or low frequencies, respectively. Therefore, as the frequency resolution increases, the time resolution decrease and vice versa (Mallat 1989). This optimal time-frequency resolution property makes the CWT technique useful for non-stationary seismic analysis.

A wavelet is defined as a function of $\psi(t) \in L^2(\mathfrak{R})$ with a zero mean, which is localised in both time and frequency. By dilating and translating the wavelet $\psi(t)$, a family of wavelets can be produced as:

$$\psi_{\sigma,\tau}(\tau) = \frac{1}{\sqrt{\sigma}} \psi\left(\frac{t-\tau}{\sigma}\right) \quad (1)$$

where σ is the dilation parameter or scale and τ is the translation parameter ($\sigma, \tau \in \mathfrak{R}$ and $\sigma \neq 0$)

The CWT is defined as the inner product of the family wavelets $\Psi_{\sigma,\tau}(t)$ with the signal of $f(t)$ which is given as:

$$F_W(\sigma, \tau) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{\sigma}} \bar{\psi}\left(\frac{t-\tau}{\sigma}\right) dt \quad (2)$$

where $\bar{\psi}$ is the complex conjugate of ψ and $F_W(\sigma, \tau)$ is the time-scale map.

The convolution integral from equation 2 can be computed in the Fourier domain. To reconstruct the function $f(t)$ from the wavelet transform, Calderon's identify (Daubechies 1992) can be used and is obtained as:

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_W(\sigma, \tau) \psi\left(\frac{t-\tau}{\sigma}\right) \frac{d\sigma}{\sigma^2} \frac{d\tau}{\sqrt{\sigma}} \quad (3)$$

$$C_\psi = 2\pi \int \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty \quad (4)$$

where $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$. The integrand in equation 4 has an integrable discontinuity at $\omega = 0$ and implies that $\int \psi(t) dt = 0$.

Procedure of the WSASW

A proposed procedure used in the WSASW is described in Figure 1 as follows.

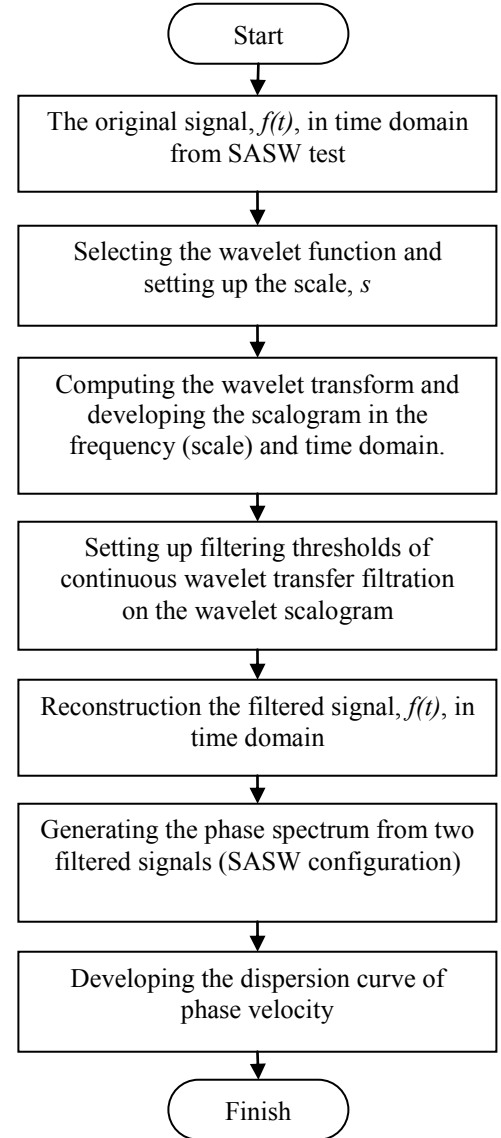


Fig 1. Flow chart of filtering procedure

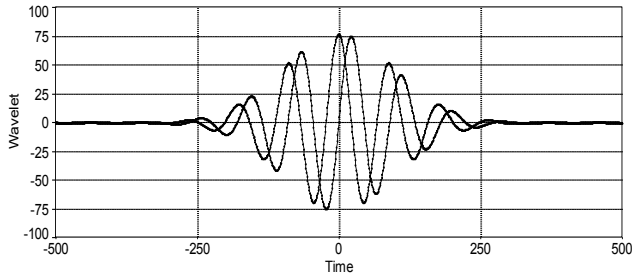
1. Select the wavelet function and a set of scale, s , to be used in the wavelet transform. The different wavelet function may influence the time and frequency resolution. In this study, a Morlet wavelet function was selected as a mother wavelet in the CWT filtering.

A commonly used wavelet in CWT is the Morlet wavelet where its shape is a Gaussian-windowed complex sinusoid. It is defined in the time and frequency domains as follows:

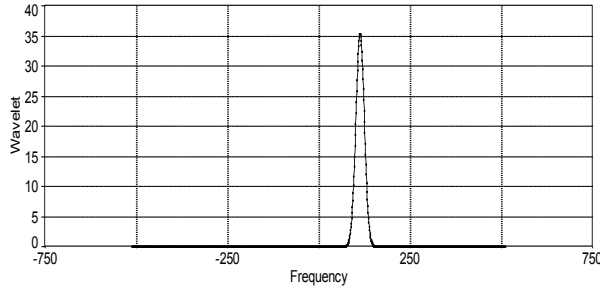
$$\Psi_0(t) = \pi^{-1/4} e^{imt} e^{-t^2/2} \quad (5)$$

$$\hat{\psi}_0(s\omega) = \pi^{-1/4} H(\omega) e^{-(s\omega-m)^2/2} \quad (6)$$

where m is the wavenumber, and H is the Heaviside function. The time and frequency domain plot of Morlet wavelet is shown in Figure 2. In Figure 2a, the Morlet wavelet is shown within an adjustable parameter m of 7 which is used in this study. This parameter can be used for an accurate signal reconstruction of seismic surface waves in low frequency. The Gaussian's second order exponential decay used in time resolution plot results in the best time localization.



(a) Time domain of real and imaginary part



(b) Frequency domain

Fig 2. Time and frequency domain plot of Morlet wavelet

2. Develop the wavelet scalogram by implementing the wavelet transform (equation 2) using computed convolution of the seismic trace with a scaled wavelet dictionary. Wavelet scale is calculated as fractional power of 2 using the formulation (Torrence & Compo 1998):

$$s_j = s_0 2^{j\delta_j}, j = 0, 1, \dots, J \quad (7)$$

$$J = \delta_j^{-1} \log_2 \left(\frac{N\delta_t}{s_0} \right) \quad (8)$$

where, s_0 is smallest resolvable scale = $2\delta_t$, δ_t is time spacing, and J is largest scale.

3. Convert the scale dependent wavelet energy spectrum (scalogram) of the signal to a frequency dependent wavelet energy spectrogram in order to compare directly with Fourier energy spectrum.
4. Perform the continuous wavelet transform (CWT) filtration on the wavelet spectrogram by obtaining the time and

frequency localization thresholds. In this study, the CWT filtration was developed by a simple truncation filter concept which only considers the passband and stopband. Threshold values in time and frequency domain are then set as the filter values between passband and stopband. It allows a straight filtering in each of the dimensions of times, frequencies and spectral energy. The noisy or unnecessary signals can be eliminated by zeroing the spectrum energy and consequently, they are fully removed when reconstructing the time domain signal. Thus, the interested spectrum of signals are to be passed when the spectrum energy is maintained in original value. A design of the CWT filtration is proposed by Rosyidi (2009) and can be written as:

$$f(s) = \begin{cases} 0, & 1 \leq s \leq F_l \\ 1, & F_l \leq s \leq F_h \\ 0, & F_h \leq s \leq N \end{cases} \quad (9)$$

$$f(u) = \begin{cases} 0, & 1 \leq u \leq T_l \\ 1, & T_l \leq u \leq T_h \\ 0, & T_h \leq u \leq N \end{cases} \quad (10)$$

5. The value of 1 means the spectrum energy is passed and the value of 0 represents the filtration criteria when the spectrum energy is set as 0.
6. Reconstruct the time series of seismic trace using equation 3 and generate the enhanced phase spectrum or the frequency response function $[H(f)]$ of the reconstructed signals which is obtained from the ratio of the linear spectra of output $[S_y(f)]$, the second geophone, over input $[S_x(f)]$, the first geophone, and its equation can be written as:

$$H(f) = \frac{S_y(f)}{S_x(f)} \quad (11)$$

7. Finally, by extracting the data of the phase angle from the phase spectrum, a composite experimental dispersion curve can be calculated by the phase difference method. The time of travel between the receivers for each frequency can be calculated by:

$$t(f) = \frac{\phi(f)}{(360f)} \quad (12)$$

where f is the frequency, $t(f)$ and $\phi(f)$ are, respectively, the travel time and the phase difference in degrees at a given frequency. The distance of the receiver (d) is a known parameter.

Therefore, the Rayleigh wave velocity, V_R or the phase velocity at a given frequency is simply obtained by:

$$V_R = \frac{d}{t(f)} \quad (13)$$

and the corresponding wavelength of the Rayleigh wave, L_R may be written as:

$$L_R(f) = \frac{V_R(f)}{f} \quad (14)$$

By repeating the procedure outlined above and using equation 12 through 14 for each frequency value, the R wave velocity corresponding to each wavelength is evaluated and the experimental dispersion curve is subsequently generated.

Shear Wave Velocity Profile

An inversion analysis was used to generate the shear wave velocity profile. In the inversion process, a profile of a set of a homogeneous layer such as pavement surface, base, subbase and subgrade layer, extending to infinity in the horizontal direction was assumed. The last layer is usually taken as a homogeneous half-space. Based on the initial profile, a theoretical dispersion curve was constructed using an automated forward modeling analysis involving 3-D dynamic stiffness matrix method (Kausel & R  esset, 1981). In the model, displacements and stresses (or traction) of the propagation of the waves on a horizontal surface can be expanded using the Fourier series in the circumferential direction and in terms of cylindrical function (Bessel, Neuman or Hankel functions) in the radial direction.

For axisymmetric loading only one Fourier series term is needed (the 0 term), and the radial and vertical displacements (U and W) can be expressed by:

$$U(r) = qR \int_{k=0}^{\infty} \bar{u} J_1(kR) J_1(kr) dk \quad (15)$$

$$W(z) = qR \int_{k=0}^{\infty} \bar{w} J_1(kR) J_0(kr) dk \quad (16)$$

where J_0 and J_1 = the zero and the first order Bessel function, k = the wave number, r = the radial distance from the source, R = the radius of the disk, q = the magnitude of the uniformly distributed load; \bar{u} and \bar{w} = functions of k for a harmonic load at the surface with wavelength $2\pi/k$. Kausel & R  esset (1981) had shown that the displacement u and w , in Equation 16, can be written as:

$$\bar{u} = \sum_{i=1}^{2n+2} u_{il} w_{il} \frac{k}{k_i(k^2 - k_i^2)} \quad (17)$$

$$\bar{w} = \sum_{i=1}^{2n+2} w_{il}^2 \frac{k}{k_i(k^2 - k_i^2)} \quad (18)$$

For a system of n layers over a half-space, u_{il} and w_{il} denote the horizontal and vertical displacements at the surface in the

i^{th} mode and can be found from the corresponding mode shape. By substituting equation 5 into 4, the integral can be evaluated analytically in closed form. This solution is particularly convenient when dealing with a large number of layers as in the case when it is desired to obtain a detailed variation of the soil properties. Subsequently, the theoretical dispersion curve generated using the 3-D model was ultimately matched to the experimental dispersion curve based on lowest root mean square (RMS) error with an optimization technique from Joh (1996).

Development of Soil Shear Modulus

The soil shear modulus profile can be obtained by linear elastic model involving the parameter of the shear wave velocity obtained from inversion process as mentioned in previous section. The soil shear modulus is calculated from the following equation (Kramer, 1996):

$$G = \frac{\gamma}{g} V_s^2 \quad (19)$$

where G = the dynamic shear modulus, V_s = the shear wave velocity, g = the gravitational acceleration; and γ = the total unit weight of the material. Nazarian & Stokoe (1986) explained that the modulus parameter of material is maximum value at a strain below about 0.001 %. In this strain range, modulus of the materials is also taken as constant.

Field procedure of surface wave measurement

In order to collect the surface wave data, the WSASW field measurement uses the common spectral analysis of surface wave method (SASW) configuration. A configuration set up on the SASW measurement is shown in Figure 3. An impact source of 8 to 12 kg was used to generate seismic waves. These waves were then received using two 1-Hz frequency natural vertical geophones. Thus, they were recorded by using a set of spectrum analyser for processing.

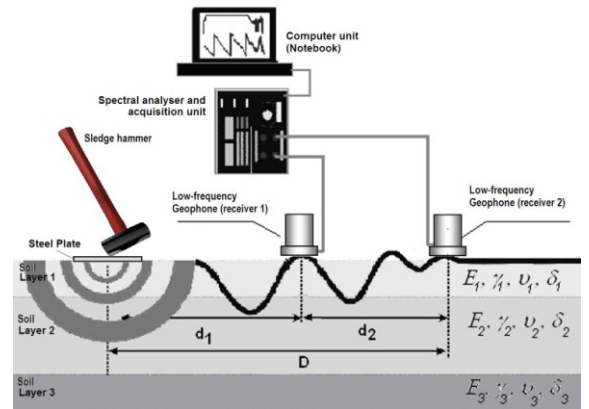


Fig 3. SASW measurement set up applied on the soil sites (Rosyidi & Taha, 2011)

Several configurations at 1, 2, 4, 8 m of the receiver and the source spacings were required in order to sample different soil depths. The configuration used in this measurement was the mid-point receiver spacings recommended by Heisey et al. (1982). In this configuration, the short receiver spacings with a high frequency source were used to sample the shallow layers of the soil profile while the larger receiver spacings with a set of low frequency sources were employed to sample the deeper layers.

RESULTS AND DISCUSSION

Site location

The SASW test was conducted at Radio Televisyen Malaysia (RTM) compound in Kelang, Malaysia. Based on two boreholes drilled to depths of 16 to 28 m, the soil type was identified as greyish clay with decayed wood at most of the soil layers of the subsoil stratum. Based on geological data, the site was classified as recent quaternary of dominantly alluvial deposits of soft marine clay with traces of organics.

Spectrum response

Figure 4 shows two example of the recorded signals from averaging multiple impacts from the SASW measurement at test site. The signals were obtained from 8 m receiver or geophone spacing. From the recorded signals, it can be recognized that higher amplitude is measured for the first mode of R-wave amplitude. It is also noted that the decreasing signal magnitude is identified as the R-wave attenuation in the soil layer which is an important characteristic for energy decrement.

The waveform of seismic signal recorded (Figure 4) is transient and non-stationary. Weak recorded signal of seismic wave particularly in channel 2 is also identified as an effect of environmental noise which maybe produced from ground noise and man-made vibrations. This means that either the input signals or behaviour of system at different moments in time was not identical. When the signals were transformed into frequency domain using FFT (fast Fourier transform), time-dependent behaviour of the seismic waves and noisy events were lost (Figure 5). The energy content of these events which are present at different times and frequency would not be picked up by conventional Fourier analysis. In other words, the conventional spectral analysis of non-stationary signal of seismic waves cannot describe the local transient event due to averaging duration of signals. It also cannot instantly separate the event of true seismic waves from noisy signals. Consequently, it is difficult to capture the correct phase information in the transfer function of both signals (Figure 6).

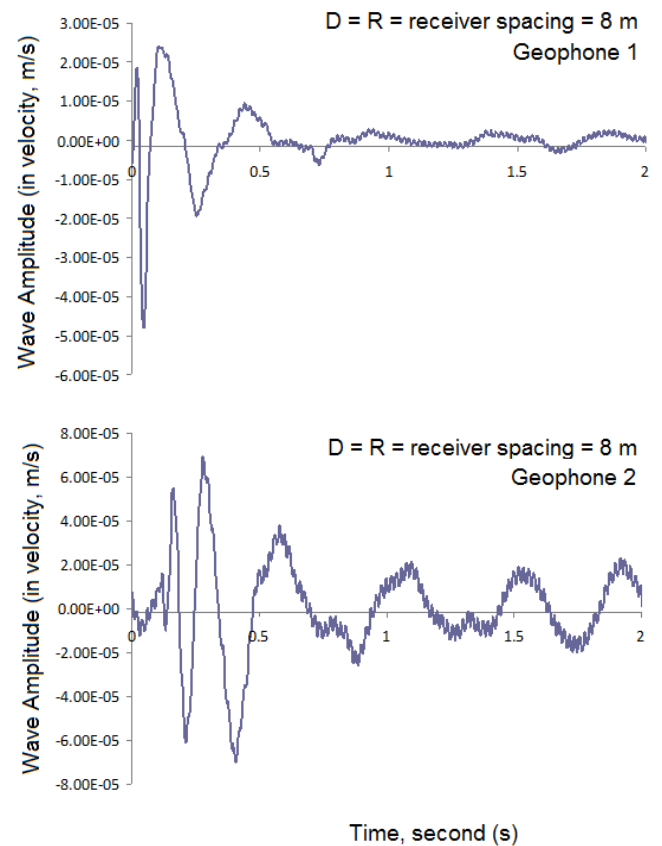


Fig 4. The time signals from 8 m receiver spacing of SASW measurement

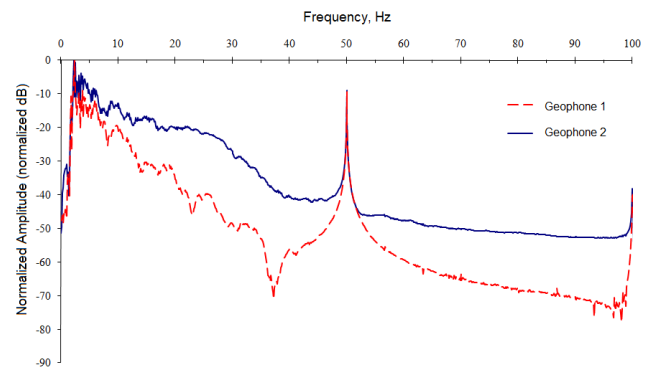


Fig 5. FFT spectrum of the time signals from 8 m receiver spacing of SASW measurement

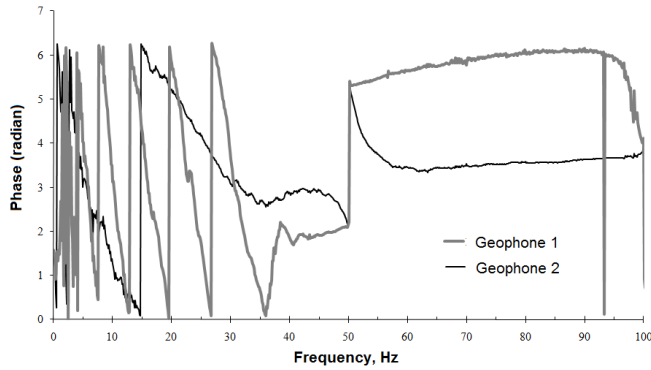


Fig 6. Phase spectrum of each time signals, i.e., geophone 1 and 2, from 8 m receiver spacing of SASW measurement

Application of CWT filtering on recorded signals

In order to enhance the pattern of non-stationary seismic wave signals from noisy signal, both signals were then transformed in time-frequency resolution by the CWT. The procedure of CWT spectrogram construction is described in Figure 1. This time and frequency analysis of CWT was employed to overcome the identification problem of spectral characteristics of signals. Figure 7 and 8 presents the CWT spectrogram of the time signal from geophone 1 and geophone 2, respectively (Figure 4), which was constructed by using a mother wavelet of Morlet.

Three main energy events at different frequency bands were clearly detected which may result in both low and high mode of seismic and noisy signals (Figure 7 and 8). It can be seen that coherent low frequency energy was found in the range of up to 2 - 10 Hz in both CWT spectrograms (event B, C, D, E). This spectrum range is clearly captured and identified as dominant noisy signals or ground rolls. Another noisy signal received during measurement was generated from the electrical devices and generator which has constantly the frequency content of 50 Hz (event F). The spectrum events of surface wave signals are recognized at event A with the frequency level of 4 to 35 Hz with arrival time of 0.012 to 0.50 s which consist of high magnitude of energy.

In order to separate the original seismic wave, the wavelet spectrogram filtration was then implemented. There are two primary ways to set the thresholds for wavelet filtering. The first is to define a region of time-frequency space. This is primarily used to isolate and reconstruct signal components. The time and frequency fields define limits in spectrogram filtering. In this study, the time and frequency range of noise signal was set as threshold of wavelet (equation 9 & 10). It means that the noisy signals are removed from the

spectrogram and only the interested seismic wave signals remain.

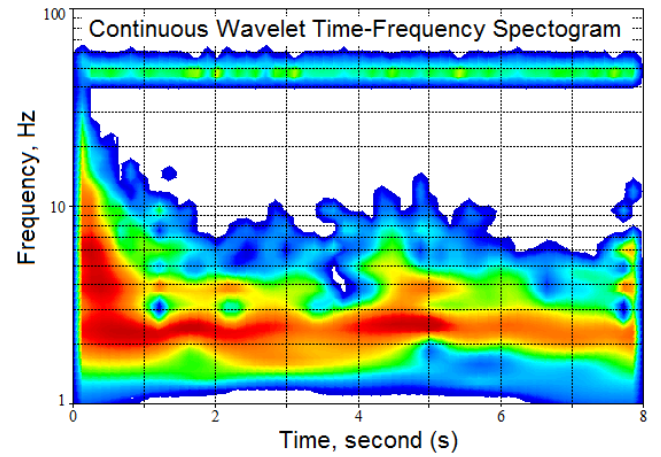


Fig 7. The CWT spectrogram for signals received by geophone 1.

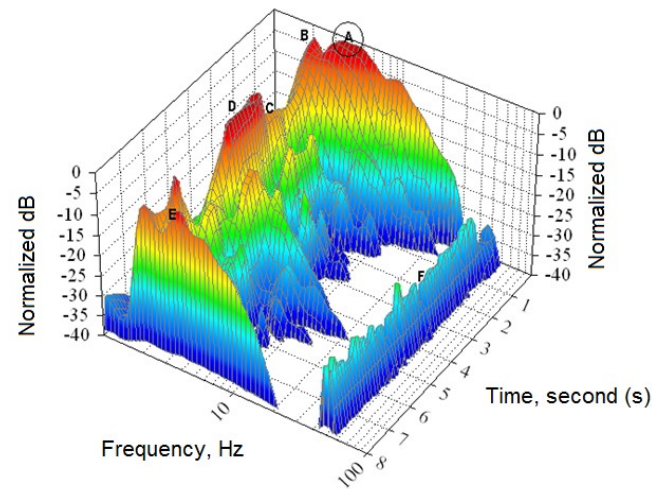


Fig 8. The CWT spectrogram for signals received by geophone 2.

Table 1 shows an example of the threshold parameters of time-frequency used in filtering criteria for the signals from 8 m receiver spacing of SASW measurement. Consequently the inverse wavelet transform returns a denoised seismic signal from the filtered spectrogram of interest. Demonstration of the wavelet analysis in denoising and reconstructing the recorded seismic signals is shown in Figure 9. Particularly for seismic signal recorded on channel 2, the reconstructed waveform of denoised signal improves the signal pattern of the seismic surface waves.

Table 1. Time and frequency threshold in WSASW

Threshold	Time (s)		Frequency (Hz)	
	T_1	T_2	F_1	F_2
Geophone 1	0.030	0.650	2.97	7.08
Geophone 2	0.002	0.440	2.75	6.05

The phase spectrum from denoised signals from the surface wave measurement was then constructed by equation 11. Compared to the phase spectrum from original signals, the enhanced phase spectrum from the CWT filtration provides the better phase information versus frequency range without noisy interference needed in the surface wave analysis (Figure 10). It shows that the CWT and wavelet filtering is an effective tool for identifying, denoising and reconstructing the noisy seismic surface waves measured on the soil profile. Finally, based on the phase different method (equation 12 – 14), a phase velocity dispersion curve from enhanced phase spectrum can be obtained. Figure 11 presents the dispersion curves obtained from CWTF compared to the original dispersion curve which is only produced from masking process.

Shear wave velocity evaluation

The actual shear wave velocity of the soil profile is produced from the inversion of the experimental dispersion curve (Figure 11). In the inversion process, a profile of a homogeneous layer extending to infinity in the horizontal direction is assumed.

The last layer is usually taken as a homogeneous half-space. Based on the initial profile, a theoretical dispersion curve is then calculated using an automated forward modeling analysis of the dynamic stiffness matrix method (Kausel & R  sset 1981). The theoretical dispersion curve is ultimately matched to the experimental dispersion curve of the lowest RMS error based on an optimization technique called the maximum likelihood method which is proposed by Joh (1996). Detail

discussion on the shear wave velocity profile generation from the SASW test is given in Rosyidi (2007).

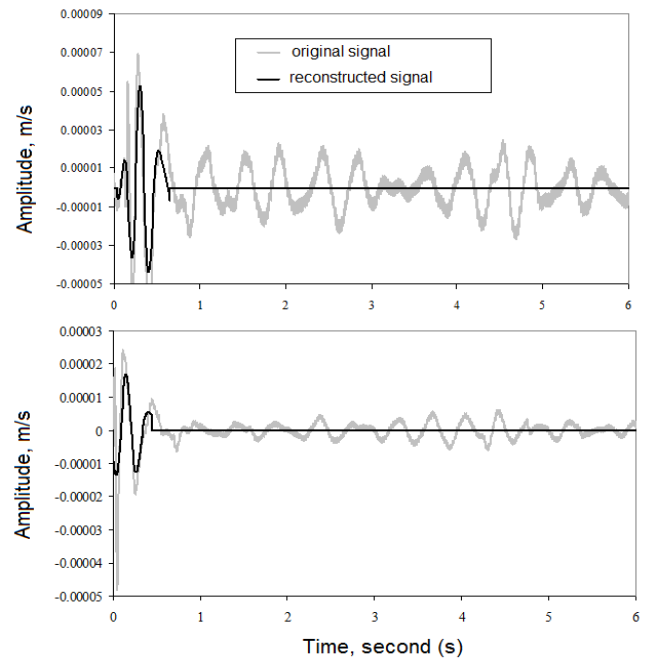


Fig 9. Reconstructed signals from the CWT filtration in the WSASW technique

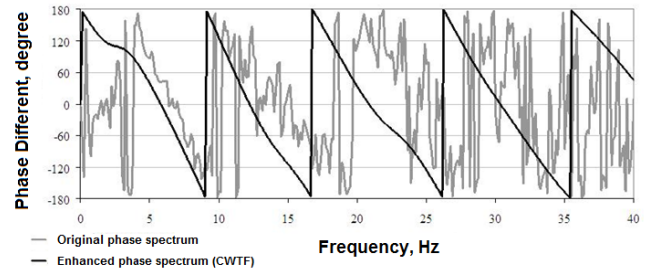


Fig 10. Phase spectrum from two signals obtained by WSASW compared to the original phase spectrum.

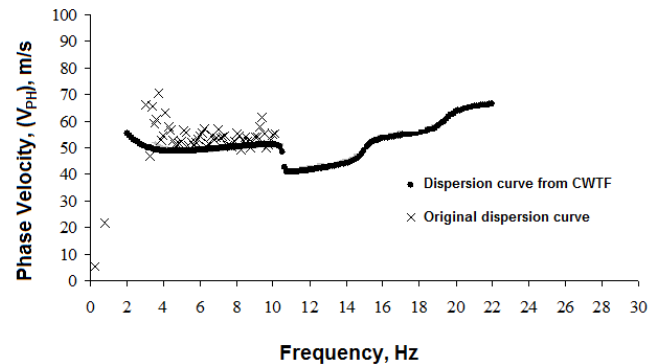


Fig 11. Comparison of dispersion curves from the WSASW and the original phase spectrum.

An example shear wave velocity profile of soil site from this study is shown in Figure 12. The average inverted shear wave velocity of soil layer for RTM Kelang test sites was found to be 54.90 m/s with a range of 38.52 to 103.53 m/s. Using the shear wave velocity parameter, the soil material in this study could be evaluated and classified as soft clay (marine clay). The result shows that the soil classification based on the shear wave velocities is also reasonably in agreement with the laboratory tests.

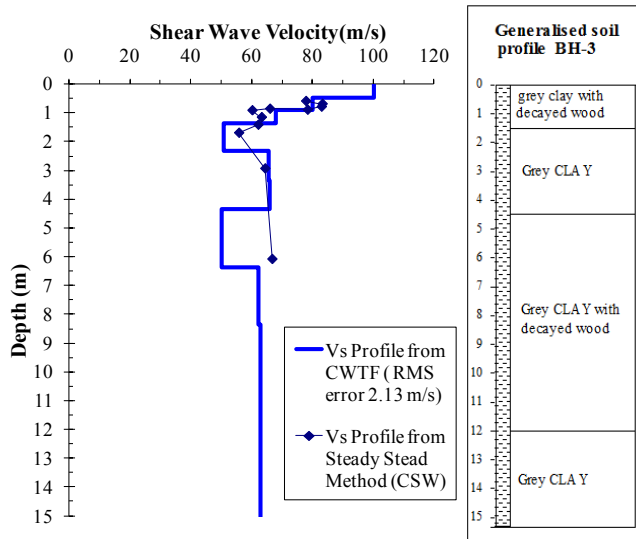


Fig 12. A shear wave velocity profile of investigated soil at RTM Kelang site and comparison with the borehole log.

As part of the validation on the results of the shear wave velocity profile obtained from this study, a steady state method or also well known as the continuous surface wave (CSW) measurement was carried out at same locations. In CSW measurement, a set of low frequency content generated from harmonic vibration source was set up in the range of 5 until 30 Hz with the fixed receiver spacing of 1 m. In this frequency level, the observed soil profile can be investigated until 6 m of depth. The comparison between a shear wave profile determined by CWTF analysis and CSW method is shown in Figure 12. This comparison shows that the shear wave velocity profile by a CWTF technique is in perfect agreement with value of the shear wave velocity determined by CSW method.

Shear modulus evaluation

Based on the shear wave velocity profile (Figure 12) and calculated using equation 15, the shear modulus profile of RTM Kelang site is given in Figure 13. The result of G is also compared with the shear modulus calculated using Hardin &

Drnevich (1972) model based on the laboratory soil parameter and the shear modulus obtained from the CSW measurement. A good agreement is obtained from these G profiles.

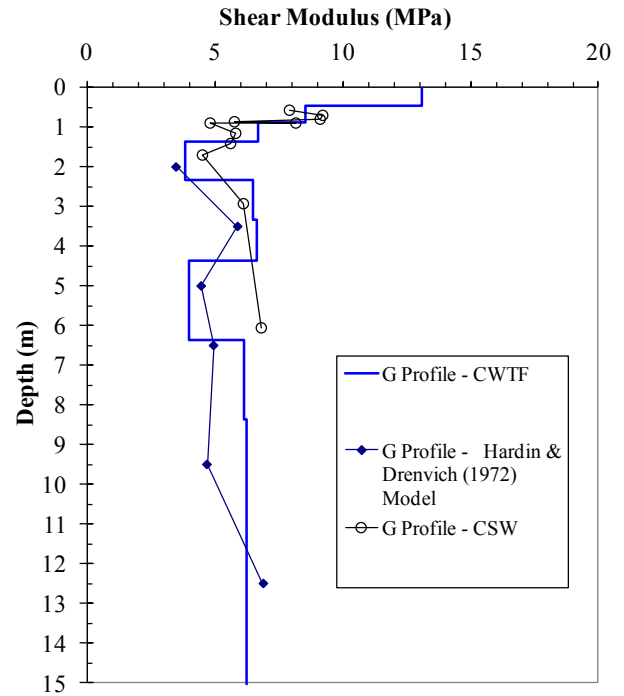


Fig 13. A shear modulus profile of investigated soil at RTM Kelang site.

CONCLUSION

A wavelet spectrogram analysis of surface waves (WSASW) is presented in this paper. The function of the mother of wavelet, Morlet, was selected based on the seismic waveform. A time-frequency analysis of the wavelet spectrogram was employed to localize the interested seismic response spectrum from the generated surface waves. Moreover, a time-frequency wavelet filtering design was implemented in this study to remove noisy distortions in the wavelet spectrogram of interest. The denoised signals of the seismic surface waves were able to be reconstructed by the inverse wavelet transform considering to the time-frequency thresholds of the interested spectrum. Based on the phase different at each frequency of the enhanced phase spectrum, an experimental dispersion curve was then developed. The WSASW is able to evaluate the reliable surface wave spectrum of the noisy signals and to develop an enhanced phase velocity dispersion curve from the surface wave measurement at a soft clay soil site as performed in this study. Good agreement was also found between the shear wave velocity and shear modulus obtained using WSASW method compared to that of the CSW method and Hardin & Drnevich (1972) model.

REFERENCES

- Daubechies, I. [1992]. “*Ten Lecturers on Wavelets*”. Society of Industrial and Applied Mathematics, Pennsylvania.
- Foufoula-Georgiou, E. & Kumar P. [1995]. “Wavelets in Geophysics: An Introduction”, in *Wavelet in Geophysics* (E. Foufoula-Georgiou & P. Kumar eds.), Academic Press, California
- Hardin, B.O. & Drnevich, V.P. [1972]. “Shear modulus and damping in soils: Measurement and parameter effects”, *Journal of Soil Mechanics and Foundations Division, ASCE* Vol. 98, No. 6, pp. 603-624.
- Heisey, J.S., Stokoe II, K.H. & Meyer, A.H. [1982]. “Moduli of pavement system from spectral analysis of surface wave”, *Transportation Research Record* 852, pp. 22-31.
- Joh, S.H. [1996]. “*Advance in interpretation and analysis technique for the spectral analysis of surface wave (SASW) measurements*”, Ph.D. Thesis, the University of Texas at Austin.
- Kausel, E. & Rössset, J.M. [1981]. “Stiffness matrices for layered soils”, *Bulletin of the Seismological Society of America*, Vol.71, No.6, pp. 1743-1761.
- Kramer, S.L. [1996]. “*Geotechnical Earthquake Engineering*”, Prentice-Hall, New Jersey.
- Nazarian, S. & Stokoe II, K.H. [1986]. “*In Situ Determination of Elastic Moduli of Pavement Systems by Spectral-Analysis-of-Surface-Wave Method (Theoretical Aspects)*”, Research Report 437-2, Center of Transportation Research, Bureau of Engineering Research, the University of Texas at Austin.
- Mallat, S. [1989]. “A theory for multiresolution signal decomposition: The wavelet representation”. *IEEE Trans. Pattern. Anal. and Mach. Intell.*, Vol.11, pp. 674-693.
- Rosyidi, S.A. [2009]. “*Wavelet analysis of surface wave for evaluation of soil dynamic properties*”. Ph.D. Thesis, Universiti Kebangsaan Malaysia, Bangi.
- Rosyidi, S.A. & Taha, M.R. [2011]. “Wavelet Spectrogram Analysis of Surface Wave Technique for Dynamic Soil Properties Measurement on Soft Marine Clay Site”, in *Seismic Waves - Research and Analysis* (Masaki Kanao ed.), InTech, Available from: <http://www.intechopen.com/books/seismic-waves-research-and-analysis/wavelet-spectrogram-analysis-of-surface-wave-technique-for-dynamic-soil-properties-measurement-on-so>.
- Stokoe II, K.H., Wright, S.G., Bay, J.A. & Rössset, J.M. [1994]. “Characterization of geotechnical sites by SASW method”. In Woods, R.D. (ed.), *Geophysical Characterization of Sites*, New Delhi: Oxford Publishers, 15-25.
- Torrence, C. & Compo, G.P. [1998]. “A practical guide to wavelet analysis”. *Bulletin of the American Meteorological Society*, Vol.79, No.1, pp. 61-78.